A Serpentine Error Diffusion Kernel with Threshold Modulation for Homogeneous Dot Distribution

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Abstract

Digital image halftoning is the process of converting a gray-scale image into its binary equivalent. Error diffusion has been one of the most popular digital halftoning techniques. The quality of binary image generated by the error diffusion method is determined by the error diffusion kernel, the sequence of processing, and the quantization scheme. The serpentine sequence of processing has been proposed for the homogeneous dot distribution. But, the binary image obtained by applying the error diffusion in serpentine sequence often exhibits vertical artifacts, especially at the gray levels of 1/4 and 3/4. In this paper, a serpentine error diffusion kernel is proposed to reduce the vertical artifacts. Also, a threshold modulation technique is proposed to further improve the homogeneity in dot distribution. The experimental results are compared with the corresponding results obtained by the conventional error diffusion techniques.

Introduction

Error diffusion has been one of the most popular digital halftoning techniques. However, the binary image obtained by the Floyd-Steinberg's¹ error diffusion algorithm exhibits various artifacts, which are visually unpleasing to human eyes. These undesirable artifacts are often divided into the so called "fingerprint" and "worm" artifacts. The former is appeared as a structured shape and is getting to be unnoticeable as printer resolution is increased. But the later, which indicates the directional artifacts in the highlight and shadow areas, are still very noticeable despite of the increased resolution.

Various literatures have been reported to improve the quality of the binary image.² Most of previous efforts proposed the modifications to the following quality factors; the values of error diffusion kernel,^{3,4} the locations of neighboring pixels,^{5,6} the sequence of processing^{5,7} and the quantization schemes.^{8,9} They can also be classified based on the following objectives; to reduce the overall artifacts,^{3,4} to enhance the edges,^{6,8,9,11} and to increase homogeneity in the highlight and shadow area.^{10,12}

Among the previous efforts, the serpentine sequence of processing, which reverses the processing direction every scan line, has been proposed to reduce the "worm" artifacts or to increase the homogeneity in dot distribution. However, the resulting binary image with the Floyd-Steinberg's kernel shows the vertical artifacts at the gray levels of 1/4 and 3/4. In this paper, a serpentine error diffusion kernel is proposed to reduce such undesirable vertical artifacts. Also, a threshold modulation technique is proposed to further improve the homogeneity in dot distribution. In the proposed method, the values of threshold are updated depending on the gray levels to generate the homogeneous dot distribution in the highlight and shadow areas.

The proposed method will be described next. Experimental results to evaluate the proposed method will be presented and compared with those based on the previous approaches.

A Serpentine Error Diffusion Kernel

Before presenting the proposed method, the Floyd-Steinberg's algorithm¹ is briefly explained first. It can be described by the following equations;

$$e(m,n) = u(m,n) - b(m,n)$$
 (1)

$$b(m,n) = 255 \qquad \text{if } u(m,n) > t(m,n)$$

= 0 else (2)

$$u(m,n) = i(m,n) + \sum_{k,l \in R} w(k,l)e(m-k,n-l)$$
(3)

where i(m,n) represents gray value and binary image b(m,n) is obtained by thresholding the "modified input" value u(m,n) with t(m,n). t(m,n) is typically constant 127 regardless of (m,n). And error e(m,n) is the difference between the modified input and binary image. *R* denotes the set of neighboring pixels for error propagation.

The gray ramp image generated by the Floyd-Steinberg's algorithm in Eqs. (1-3) with the conventional raster scanning sequence is shown in Figure 1. The threshold was constant 127. In Figure 1, the "worm artifacts" or non-homogeneous dot distributions in the highlight and shadow areas are noticeable. As an effort to reduce the "worm artifact" shown in Figure 1, a serpentine sequence of processing has been proposed. It reverses the processing direction every scan line. It has been theoretically proven that the error diffusion with the serpentine sequence can generate homogeneous dot distribution.⁷ Figure 2 shows the binary ramp image generated by processing the ramp image in the serpentine sequence with the Floyd-Steinberg's kernel. Again, constant threshold 127 is used. As shown in Figure 2, the homogeneity of dot distribution in the highlight and shadow areas has been improved but the vertical artifacts are visible at the gray levels of 1/4 and 3/4.

In this paper, the following error diffusion kernel is proposed for the serpentine sequence of processing to reduce the vertical artifacts.

$$\begin{bmatrix} w(2,1) & w(2,0) \\ w(1,1) & w(1,0) \\ w(0,1) \end{bmatrix} = \begin{bmatrix} 2/16 & 3/16 \\ 3/16 & 4/16 \\ 4/16 & * \end{bmatrix}$$
(4)

The binary ramp image generated by processing the ramp image in the serpentine sequence with the proposed kernel in Eq. (4) is shown in Figure 5. The threshold was 127. The vertical artifacts appeared on Figure 2 have been considerably reduced in Figure 3. However, Figure 3 lacks the homogeneity of dot distribution. In order to further improve the homogeneity of dot distribution, a threshold modulation technique is proposed in this paper and will be explained next.





Figure 3. Serpentine Scan with the Proposed Error Diffusion Kernel and Constant Threshold 127

Proposed Threshold Modulation Technique

As shown in Figure 3, the serpentine raster scan with the proposed error diffusion kernel can reduce the unpleasing vertical artifact. But, there is room for improvement in the homogeneity of dot distributions. In this paper, a threshold modulation method is proposed for the homogeneous dot distributions in the highlight and shadow area. The proposed technique can be summarized as follows; when a black dot is printed in the highlight area, the values of threshold for the neighboring pixels are reduced to prevent from printing another black dot in the vicinity of the printed black dot. The degree of reduction depends on the gray level of the pixel. If the current pixel is halftoned to be white, the threshold values for the neighboring pixels are increased to gradually restore the chance of printing another black dot. Again, the amount of addition depends on the gray level of pixel. Similar procedure is applied to update the threshold values in the shadow areas. The details of the proposed technique are described next.

First, the threshold value t(m,n) is initially set to 127 for all (m,n). In the highlight area, i.e., i(m,n) > 127, the threshold values are updated according to the following equations;

$$t(m + p, n + q) + = Tf_1(p,q) \times t(m,n)$$
 if $b(m,n) = 255$ (5)

$$t(m+p,n+q) + = Tf_2 \qquad \text{else} \qquad (6)$$

where t(m + p, n + q) denotes the threshold value for the neighbor with displacement (p,q). $Tf_1(p,q)$ and Tf_2 are "threshold modulation factors" to determine the degree of addition and reduction, respectively. $Tf_1(p,q)$ determines the restoring speed of threshold value and Tf_2 acts like an impulse that prevents from printing another black dot next to each other. Threshold modulation factors $Tf_1(p,q)$ and Tf_2 are defined as follow;

$$Tf_1(p,q) = \rho(p,q) \times \left[\frac{|i(m,n) - 127.5|}{127.5}\right]^s$$
(7)

$$Tf_2 = -\sigma \times \left[\frac{|i(m,n) - 127.5|}{127.5} \right]^t$$
 (8)

where $\rho(p,q)$ is a constant depending on the displacement (p,q). σ ,s, and t are constants. It should be cautioned that the values of $Tf_1(p,q)$ and Tf_2 need to be selected carefully. If $Tf_1(p,q)$ is too high, it does not preserve the local gray average and results in slow response in the transition area. If too small, the dot distribution does not appear homogeneous.

In the shadow areas, i.e., $i(m,n) \le 127$, the threshold values are updated according to the following equations;

$$t(m+p,n+q) = Tf_1(p,q) \times t(m,n) \quad \text{if } b(m,n) = 0 \tag{9}$$

$$t(m+p,n+q) = Tf_2 \qquad \text{else} \qquad (10)$$

Eq. (9) is to decrease the threshold values for neighboring pixels to gradually restore the chance of printing another black dot. Eq. (10) is to reduce the chance of the white pixels appeared next to each other by increasing the threshold values.

Experimental Results

In order to evaluate the performance of the proposed error diffusion kernel and the threshold modulation technique, the gray ramp image is binarized first. In Eq. (5-10), the threshold values of two neighbors are considered for the threshold modulation to reduce computational cost. They are the threshold values of next pixel in current scan line, i.e., t(m,n+1) or t(m,n-1) depending on the current direction of processing, and pixel be-

low current scan line, t(m+1,n). In Eq. (7), $\rho(p,q) = 3.2$ when (p,q) = (0,1) or(0,-1), $\rho(p,q) = 2.4$ when (p,q) = (1,0). In Eq. (7-8), the values of constants are $\sigma = 70000$, s = 2, t = 30. When i(m,n) = 0 or 255, the values of $Tf_1(p,q)$ and Tf_2 are chosen to be 0.

Figure 4 shows the binary ramp image obtained by the proposed method, i.e., by applying the serpentine error diffusion kernel in Eq. (4) and the threshold modulation in Eqs. (5-10). Compared to the image in Figure 3 which employs the proposed kernel only, it can be said that the proposed threshold modulation technique improves the homogeneity of dot distributions in the highlight and shadow areas.

In order to compare the performance of the proposed method with those of the existing works, Eschbach's¹⁰ and Fan's⁵ algorithms are applied to the same gray ramp image. Figures 5 and 6 show the resulting images by Eschbach's¹⁰ and Fan's⁵ algorithms, respectively. In Eschbach's¹⁰ algorithm, Floyd-Steinberg's kernel is employed with the standard raster sequence. Figure 6 is obtained by applying Fan's⁵ kernel of 3 neighbors and processing the same line twice in opposite directions. Compared to Figures 5 and 6, the image in Figure 4 shows faster responses in the both extreme ends of the highlight and shadow areas.





Figure 6. Gray Ramp Result of Fan's Algorithms

In order to evaluate the differences in dot distributions in detail, a constant gray image having four gray levels 246, 248, 250, and 252 is generated. The resulting binary images are shown in Figures 7-9. The background is a constant gray level of 135. Figures 7-9 are obtained by Fan's,⁵ Eschbach's,¹⁰ and the proposed method, respectively. Compared to Figure 8 and 9, Figure 7 lacks in the homogeneity. Although Figure 7 and 8 shows the horizontally or vertically distributed dot patterns, the image by the proposed method in Figure 9 has the dot patterns evenly distributed at 45°. Also, compared to Figure 8 and 9, Figure 9 exhibits faster responses in the transition areas.

Next, a portrait image which has wide highlight area is chosen for halftoning. Figures 10-12 shows the binary images by Floyd-Steinberg's algorithm,¹ Eschbach's algorithm¹⁰ and the proposed method, respectively. They are printed at 150 dpi resolution. Compared to Figures 10 and 11, Figure 12 shows less structured artifacts in the face area.



Figure 7. Constant gray result of Fan's algorithm⁵



Figure 8. Constant gray result of Eschbach's algorithm¹⁰



Figure 9. Constant gray result of the proposed algorithm

Conclusions

The halftoned image obtained by applying the Floyd-Steinberg's error diffusion algorithm in the serpentine sequence exhibits relatively-homogeneous dot distributions but shows unpleasing vertically structured artifacts. This paper presents a serpentine error diffusion kernel and a threshold modulation technique. Various experiments are performed to examine the effect of the pro-



Figure 10. Result of applying the Floyd-Steinberg's error diffusion algorithm,



Figure 11. Result of applying Eschbach's algorithm.

posed algorithm. According to the experimental results, it can be said that the proposed serpentine error diffusion kernel reduces the vertical artifacts. The proposed threshold modulation technique generates homogeneous dot distributions in the highlight and shadow areas and shows fast response in the transition area.



Figure 12. Result of applying the proposed algorithm.

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